**3-D co-geometry**

**1.** Find the point of intersection of the plane and the line

.

Find also the angle between the line and the plane.

Substitute the line in the plane,

, ,

The direction of the given straight line is

The normal of the plane is

Angle between the line and the plane =

**2.** The position vectors of points P and Q referred to an origin O are given by

respectively. Show that the cosine of is equal to .

Hence, or otherwise, find the position vector of M on such that is perpendicular to .

Since M on , , where m is a scalar.

Since ,

**3.** Given two lines

Determine whether these lines are parallel, intersect or skewed.

Rewrite in parametric form,

Direction of :

Direction of :

Since for any scalar k, and are not parallel.

Equate the corresponding x, y z values,

There is no intersection point for and .

∴ and are skewed.

**4.** , are two planes.

is the origin and the point has coordinates .

**(a)** Verify that the vector is parallel to both and .

**(b)** Find the vector equation of the plane which passes through and is perpendicular to both

and .

**(c)** Find the coordinates of one point common to and and hence, find the Cartesian equation of the line of intersection of and .

**(a)** The normal to the plane is .

The normal to the plane is .

Since

∴ The vector is parallel to both and .

**(b)**

Since is a vector perpendicular to the plane made by the normals of and ,

is the normal of the plane perpendicular to both and .

Let

Let the vector equation of the plane which passes through and is perpendicular to both

and is and be a variable point on .

The required vector equation is

**(c)** We can find a common point by putting in and .

We get .

Solve we get

Then is a point common to and .

in (b) is a vector parallel to the line of intersection of and .

The required equation is .

**5.** The points A, B, C and D have positive vectors, relative to the origin O, given by

, where a is a constant.

Given OA is perpendicular to OB,

**(a)** find the value of a and ,

**(b)** show that **OA** is normal to the plane OBC,

**(c)** find an equation of the plane through D parallel to the plane OBC and, hence, find the position

vector of the point of intersection of this plane and the line AC.

**(a)** ,

**(b)** which is the normal of the plane OBC.

Since

Therefore OA is normal to the plane OBC.

**(c)** Since is on the plane, OBC: .

Let the required plane be

D is on , therefore .

Solve we get

or

The line AC must pass through .

It cuts the plane , therefore

∴Intersection point

**6.** Let the equations of two planes be .

**(a)** Find the acute angle between and , giving your answer to the nearest .

**(b)** Determine the length of the projection of the vector to .

**(c)** Find the equation of the plane which is perpendicular to both and passes though thepoint .

**(a)** The angle between two intersecting planes is the angle between their normal vectors.

are the normals of respectively.

, to the nearest .

**(b)** Let ,

∴Length of the projection of the vector to

**(c)**

Since is a vector perpendicular to the plane made by the normals of and ,

is the normal of the plane perpendicular to both and .

Let

Let the vector equation of the plane which passes through and is perpendicular to both

and is and be a variable point on .

The required vector equation is

**7.** are six points in three dimensional space. The points A, B and C lie on the plane , whereas the points D, E and F lie on the plane .

A straight line passes through the points E and F.

**(a)** Determine whether and are perpendicular vectors.

**(b)** Find the Cartesian equation of the plane .  
 **(c)** Find the equation of line in vector form and in parametric form.

**(d)** Find the coordinates of the point of intersection of the line and the plane .

**(e)** Find the Cartesian equation of the straight line passes through the point

and perpendicular to the plane .

**(f)** Find the position vector of the point of intersection between the lines and .

**(g)** Find the acute angle between the plane and the plane .

**(a)** , .

and are not perpendicular vectors.

**(b)** . which is the normal of the plane .  
 Since is on the plane, the Cartesian equation of the plane is

**(c)**

Since is on , equation of the line is

In parametric form,

**(d)** Put in ,

The point of intersection of the line and the plane is

or

**(e)** The normal of the plane is which is the direction of .

The Cartesian equation of the straight line passes through the point

and perpendicular to the plane is

**(f)** Substitute in , we get

The required point is .

The position vector of the point of intersection between the lines and is

**(g)**

=

∴The acute angle between the plane and the plane is

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