**3-D co-geometry**

**1.** Find the point of intersection of the plane and the line

 .

 Find also the angle between the line and the plane.

 Substitute the line in the plane,

 , ,

The direction of the given straight line is

The normal of the plane is

 Angle between the line and the plane =

**2.** The position vectors of points P and Q referred to an origin O are given by

 respectively. Show that the cosine of is equal to .

 Hence, or otherwise, find the position vector of M on such that is perpendicular to .

 Since M on , , where m is a scalar.

 Since ,

**3.** Given two lines

 Determine whether these lines are parallel, intersect or skewed.

Rewrite in parametric form,

 Direction of :

 Direction of :

 Since for any scalar k, and are not parallel.

 Equate the corresponding x, y z values,

 There is no intersection point for and .

 ∴ and are skewed.

**4.** , are two planes.

 is the origin and the point has coordinates .

 **(a)** Verify that the vector is parallel to both and .

 **(b)** Find the vector equation of the plane which passes through and is perpendicular to both

  and .

 **(c)** Find the coordinates of one point common to and and hence, find the Cartesian equation of the line of intersection of and .

 **(a)** The normal to the plane is .

 The normal to the plane is .

Since

 ∴ The vector is parallel to both and .

 **(b)**

 Since is a vector perpendicular to the plane made by the normals of and ,

 is the normal of the plane perpendicular to both and .

 Let

 Let the vector equation of the plane which passes through and is perpendicular to both

 and is and be a variable point on .

 The required vector equation is

 **(c)** We can find a common point by putting in and .

 We get .

 Solve we get

 Then is a point common to and .

 in (b) is a vector parallel to the line of intersection of and .

 The required equation is .

**5.** The points A, B, C and D have positive vectors, relative to the origin O, given by

 , where a is a constant.

 Given OA is perpendicular to OB,

 **(a)** find the value of a and ,

 **(b)** show that **OA** is normal to the plane OBC,

 **(c)** find an equation of the plane through D parallel to the plane OBC and, hence, find the position

 vector of the point of intersection of this plane and the line AC.

**(a)** ,

**(b)** which is the normal of the plane OBC.

 Since

 Therefore OA is normal to the plane OBC.

**(c)** Since is on the plane, OBC: .

 Let the required plane be

 D is on , therefore .

 Solve we get

 or

 The line AC must pass through .

 It cuts the plane , therefore

 ∴Intersection point

**6.** Let the equations of two planes be .

 **(a)** Find the acute angle between and , giving your answer to the nearest .

 **(b)** Determine the length of the projection of the vector to .

 **(c)** Find the equation of the plane which is perpendicular to both and passes though thepoint .

 **(a)** The angle between two intersecting planes is the angle between their normal vectors.

 are the normals of respectively.

 , to the nearest .

 **(b)** Let ,

 ∴Length of the projection of the vector to

 **(c)**

 Since is a vector perpendicular to the plane made by the normals of and ,

 is the normal of the plane perpendicular to both and .

 Let

 Let the vector equation of the plane which passes through and is perpendicular to both

 and is and be a variable point on .

 The required vector equation is

**7.** are six points in three dimensional space. The points A, B and C lie on the plane , whereas the points D, E and F lie on the plane .

 A straight line passes through the points E and F.

 **(a)** Determine whether and are perpendicular vectors.

 **(b)** Find the Cartesian equation of the plane .
 **(c)** Find the equation of line in vector form and in parametric form.

 **(d)** Find the coordinates of the point of intersection of the line and the plane .

 **(e)** Find the Cartesian equation of the straight line passes through the point

 and perpendicular to the plane .

 **(f)** Find the position vector of the point of intersection between the lines and .

 **(g)** Find the acute angle between the plane and the plane .

 **(a)** , .

 and are not perpendicular vectors.

 **(b)** . which is the normal of the plane .
 Since is on the plane, the Cartesian equation of the plane is

 **(c)**

 Since is on , equation of the line is

 In parametric form,

 **(d)** Put in ,

 The point of intersection of the line and the plane is

 or

 **(e)** The normal of the plane is which is the direction of .

 The Cartesian equation of the straight line passes through the point

 and perpendicular to the plane is

 **(f)** Substitute in , we get

 The required point is .

 The position vector of the point of intersection between the lines and is

 **(g)**

 =

 ∴The acute angle between the plane and the plane is

**Yue Kwok Choy**

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